Check-Up

29

Knowledge of metric units of area and solid volume

a) The area of a sheet of A0 paper is 1 m^2. Given that the length is about 1189 mm, use a calculator to find the width. A1 paper is half of a sheet of A0 paper. So what are the length and width of a sheet of A1? Now check that its area is 0.5 m^2.

b) The budget for a new school playing-field allows for a total area of 0.6 ha (0.6 of a hectare). Suggest some possible lengths and widths for a rectangular field of this size.

c) Without using a calculator, find how many rectangular boxes (cuboids), 25 cm by 50 cm by 20 cm, would fit into a metre-cube? What is the volume in m^3 of one of these boxes?

Answers to check-up 29

a) A0 paper is approximately 1189 mm by 841 mm; A1 paper is about 841 mm by 594.5 mm.

b) 100 m by 60 m, 80 m by 75 m, 125 m by 48 m…

c) 40 boxes; the volume of one box is 0.025 m^3.

Discussion and explanation of check-up 29

The area of a rectangle is given by multiplying together the lengths of its two sides. If the two lengths are given in metres (e.g. 3 m by 4 m), then the area will be in square metres (12 square metres). The symbol for 'square metre' is m^2. My advice is to read this as 'square metre', not as 'metres square'. This language can otherwise cause confusion. For example, if you say 'five metres square' do you mean an area of 5 m^2 or a square of side 5 metres (which has an area of 25 m^2!). I would avoid the ambiguity by saying that a 5-metre square has an area of '25 square metres', while writing the area as 25 m^2.

For (a), since the area of the A0 paper is given in square metres (1 m^2) we start by writing the length in metres, as 1.189 m. Then the width is given by 1 ÷ 1.189 = 0.8410429 m (that's about 841 mm or 84.1 cm). The width of A0 becomes the length of A1. The width of A1 is half the length of A0, about 594.5 mm, or 0.5945 m. The area of A1 is therefore 0.8410429 0.5945 = 0.5 m^2, (approximately) or half a square metre. This relationship continues with other paper sizes: A2 (about 594.5 mm by 420.5 mm) has an area of 0.25 m^2 (a quarter of a square metre), and so on.

Units for larger areas include the *are (a)* and the *hectare (ha)*. An are is 100 m^2. Think of a 10-metre square (10 m 10 m), which is about the size of a classroom. A hectare is 100 ares (the prefix *hecto* means a hundred), or 10 000 m^2. Think of a square field 100 m by 100 m. So an area of 0.6 ha is 6000 m^2. That could be a rectangular field with sides 100 m by 60 m, and so on. If you have a rural background you may have some sense of the size of an acre. So, just for reference, I might mention that an area of 200 m by 20 m is about an acre. That's 4000 m^2 or 0.4 ha.

One cubic metre (1 m^3) is the volume of a metre-cube; that's a cube with sides of 100 cm. This has a total volume of 100 100 100 = 1 000 000 cm^3

(a million cubic centimetres). In (c), assuming the width of the boxes is 25 cm, we could fit a row of 4 of these along one edge of a metre-cube. If each box is 50 cm long, we would then need 2 of these rows to make a layer of boxes covering the bottom of the metre-cube. That's 8 boxes in each layer. The height of each layer is 20 cm, so we could fit 5 layers into the metre-cube, giving us a total of 40 boxes. Writing the sides in metres, each box has a volume of 0.25 0.5 0.2 = 0.025 m^3. (Note that $0.025 = \frac{1}{40}$.)

Summary of key ideas

- The area of a rectangle is found by multiplying together the lengths of the sides; but make sure these lengths are given in the same units.
- If the lengths are in metres (m), the area will be in square metres (m^2); if the lengths are in centimetres (cm), the area will be in square metres (cm^2); and so on.
- 1 m^2 = 10 000 cm^2.
- The volume of a rectangular box (also called a cuboid) is found by multiplying together the lengths of the three sides; again make sure the same units are used for all three sides.
- If the lengths are in metres (m), the volume will be in cubic metres (m^3); if the lengths are in centimetres (cm), the volume will be in cubic centimetres (cm^3); and so on.
- 1 m^3 = 1 000 000 cm^3.
- A hectare is 10 000 m^2, which is the area of a square field 100 m 100 m.

Further practice

29.1 Measure the lengths of the sides of a sheet of A5 paper to the nearest millimetre. Check that these lengths give an area of about $\frac{1}{32}$ of a square metre.

29.2 A 1-litre carton of milk is a cuboid with sides approximately 17.5 cm, 9.5 cm and 6.5 cm. What is the volume in cubic metres of one of these cartons?

29.3 An acre is about 0.4 ha. About how many acres is a hectare? Suggest some possible lengths for the sides of a rectangular field with an area of 0.4 ha.